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NAGPUR UNIVERSITY, NAGPUR

As per National Education Policy-2020

Curriculum Framework for M.Sc. Mathematics Four Semester (Two Years) Post Graduate (CBCS) Degree Course

M.Sc. Semester III and IV
Implemented with effect from the academic year 2024-25

PROGRAM: M. Sc. Mathematics (CBCS)

Program Outcome:

PO1. Critical Thinking: Take informed actions after identifying the assumptions that frame our thinking and actions, checking out the degree to which these assumptions are accurate and valid, and looking at our ideas and decisions (intellectual, organizational, and personal) from different perspectives.

PO2. Problem Solving: Solve problems from the disciplines of concern using the knowledge, skills and attitudes acquired from mathematics/ sciences/social sciences/humanities.

PO3. Effective Communication: Speak, read, write and listen clearly in person and through electronic media in English and in one Indian language, and make meaning of the world by connecting people, ideas, books, media and technology.

PO4. Individual and Team Work: Function effectively as an individual, and as a member or leader in diverse teams, and in wide variety of settings.

PO5. Ethics: Understand multiple value systems including your own, the moral dimensions of your decisions, and accept responsibility for them.

PO6. Environment and sustainability: Understand the impact of technology and business practices in societal and environmental contexts, and sustainable development.

PO7. Self-directed and life-long learning: Demonstrate the ability to engage in independent and life-long learning in the broadest context socio-technological changes.

PO8. Design/Development of Solutions: Design solutions for complex science problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO9. Computational Thinking: Understand data-based reasoning through translation of data into abstract concepts using computing technology-based tools.

PO10. Effective Citizenship: Demonstrate empathetic social concern and equity centered national development, and the ability to act with an informed awareness of issues and participate in civic life through volunteering.

PO11. Global Perspective: Understand the economic, social and ecological connections that link the world's nations and people.

PO12. Aesthetic Engagement: Demonstrate and master the ability to engage with the arts and draw meaning and value from artistic expression that integrates the intuitive dimensions of participation in the arts with broader social, cultural and theoretical frameworks.

Program Specific Outcome:

PSO1: **Rational Thinking**: Students be able to formulate and develop Mathematical arguments in a logical manner to unravel the gist hidden in the problem at hand.

PSO2: **Problem solving ability**: Student should be able to think in a critical manner to process the data, and develop Mathematical problem-solving ability.

PSO3: **Revisiting the question**: Students should be able to recall basic facts, important milestones, discoveries in Mathematics and inculcate habit of rational thinking by which the problem at hand can be revisited, time and again, that helps in solving it.

PSO4: **Analytical ability**: In the growing field of research, it is necessary for students to learn to use some packages like La-Tex, Matlab, Scilab, Mathematica, Maxima, etc, so that analytical tools be available to investigate the functions, problems through graphs, programming, etc.

PSO5: **Numerical Ability**: Using packages, students can make programs to solve some problems of which exact solutions are not available, using tools of Numerical analysis.

PSO6: **Simulation Ability**: The problems that cannot be solved directly, can at times be solved through techniques of simulation by honors/research students.

PSO7: **Research**: Students thus motivated would prepare themselves for research studies in Mathematics and related fields.

PSO8: **Application**: Student will be able to apply their skills and knowledge in Mathematics to various fields of studies including, science, engineering, commerce and management etc.

<table>
<tr><th colspan="2">M.Sc. Semester III (MATHEMATICS)</th></tr>
<tr><td>Semester III
Paper I-M10
Code: MMT3T10

DSC (Core)
Credits-4

No. of hours 60</td><td>Course Title: COMPLEX ANALYSIS

Course Outcomes:

CO1: Foundational Knowledge: Students will be able to learn about complex number. They will get the knowledge about complex number in polar coordinate as well.

CO2: Elementary Skills: Students will be able to learn the Mobius Transformation, they come to know about the Analytic Function and Elementary Properties, Power Series.

CO3: Basic Analytic skills: The Objective of this Course is to study the topic like Cauchy-Riemann Equations, Poles and Residues, Mobius Transformation.

CO4: Application: By applying the principles of basic tools through the course curriculum, Student can solve variety of practical problems.</td></tr>
<tr><td rowspan="3"></td><td>Syllabus for M.Sc. Semester – III
Paper -M10: Complex Analysis
MMT3T10</td></tr>
<tr><td>Unit I – The Complex Number System:
The complex plane, Polar representation and roots of complex numbers, The extended plane and its spherical representation (Ch-I Section $3, $4, $6), Power series, Analytic Functions (Ch-III Section $1, $2).

Unit II – Elementary Properties and Examples of Analytic Functions:
Analytic functions as mapping, Mobius transformation (Ch-III Section $3), Power series representation of analytic functions, zeros of an analytic function, index of a closed curve (Ch-IV Section $2, $3, $4).

Unit III – Complex Integration:
Cauchy’s theorem (Statement Only) and Integral Formula, The homotopic version of Cauchy’s Theorem and simple connectivity (Definition & Statement Only), Counting zeros; the Open Mapping Theorem, Goursat’s Theorem (Ch-IV Section $5, $6, $7, $8). Classification of singularities, Residues, The Argument Principle (Ch-V Section $1, $2, $3).

Unit IV – The Maximum Modulus Theorem:
The maximum principle. Schwarz’s lemma. Convex functions and Hadamards three circles theorem. Phragmen-Lindelöf theorem (Ch-VI).</td></tr>
<tr><td>Text Book:
Functions of one complex variable: John B. Conway, Second edition, Springer international Student Edition.</td></tr>
</table>

	Reference Books: 1. Complex Analysis, L.V. Ahlfors. Mc-Graw Hill, 1966. **2.** R.V. Churchill and J.W. Brown, Complex Variables and Applications (eighth edition), McGraw Hill Publication

Semester III **Paper II-M11** **Code: MMT3T11** **DSC (Core)** **Credits-4** **No. of hours 60**	Course Title: **FUNCTIONAL ANALYSIS** **Course Outcomes:** CO1: ***Foundational Knowledge****: Students will be able to update their basics knowledge in spaces, subspace, sequences, continuity and Normed space* CO2: ***Elementary Skills****: Students will able to understand the importance of uniform convergence, inner product space, orthogonal sets and orthonormal sets.* CO3: ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytical skills and to help sharpen the student's understanding of the mathematical structure of the subject.* CO4: ***Application****: By applying the principles of basic tools through the course curriculum, students can solve a variety of practical problems in science and engineering.*
	Syllabus for M.Sc. Semester – III **Paper II - M11: Functional Analysis** **MMT3T11**
	Unit I Normed spaces, Banach spaces, Further properties of normed spaces. Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded and continuous linear operators.
	Unit II Linear functionals. Normed spaces of operators. Dual spaces. Inner product space. Hilbert space. Further properties of inner product spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Total orthonormal sets and sequences.
	Unit III Representation of functionals on Hilbert spaces. Hilbert adjoint operators, self-adjoint, unitary and normal operators. Hahn-Banach Theorem, Hahn-Banach Theorem for complex vector spaces and normed spaces. Reflexive spaces.
	Unit IV Category theorem, Uniform boundedness theorem, strong and weak convergence, Convergence of sequences of operators and functionals. Open mapping theorem, Closed linear operators and closed graph theorem.

	Text Book: Introductory Functional Analysis with Applications by E. Kreyszig, John Wiley and Sons.
	Reference Books: **1.** Introduction to Functional Analysis by A.E. Taylor and D.C. Lay, John Wiley and Sons. **2.** Introduction to Topology and Modern Analysis: G.F. Simmons, Mc Graw Hill

Semester III **Paper III-M12** **Code: MMT3T12** **DSC (Core)** **Credits-4** **No. of hours 60**	Course Title: **ADVANCE MATHEMATICAL METHODS** **Course Outcomes:** CO1: ***Application in Engineering:*** *By applying the principles of basic formulae of Fourier series through the course curriculum, students can solve a variety of logical problems in science and engineering.* CO2: ***Application in physics to Solve Boundary value Problem:*** *Students will be able to understand the importance of Laplace Transform to solve physical and Electrical Problem.* CO3: ***Application to Solve Boundary Value Problem:*** *The main outcome of the course is to equip students with necessary basic analytic skills for problem solving on Partial Differential Equation.* CO4: ***Application in Engineering****: By applying the principles of basic formulae of Z-Transform through the course curriculum, students can solve a variety of logical problems in science and Electrical engineering.*
	Syllabus for M.Sc. Semester – III **Paper III- M12: Advance Mathematical Methods** **MMT3T12**
	Unit I – Fourier Series: Periodic Functions, Fourier series, Dirichlet's condition for a Fourier Series, Determination of Fourier Coefficients (Euler's formulae), Functions defined in two or more sub-ranges, Discontinuous functions, Even and Odd functions, Half range series, Change of Intervals and functions having arbitrary period, Parseval's formula, Fourier Series in Complex form, Practical Harmonic Analysis

	Unit II - Laplace Transform: Introduction, Properties of Laplace Transform: Laplace Transform of the derivative of f(t), Laplace Transform of Derivative of Order n, Laplace Transform of integral of $f(t)$, Laplace Transforms of $t^n f(t)$, Laplace Transform of $\left\{\frac{f(t)}{t}\right\}$. Unit Step function, Second Shifting Theorem , Unit Impulse function, Periodic functions, Convolution Theorem, Evalution of Integral,Inverse Laplace Transform, Properties of Inverse Laplace Transform : Multiplication by s, Division by s, First and second shifting Property, Inverse Laplace Transform of Derivatives, Inverse Laplace Transform of Integrals, Partial fraction Method. Solution of Differential equation by Laplace Transform, Solution of Simultanious Differential Equation by Laplace Transform, Methods to find out residues: Inversion formula for the Laplace Transform, Heaviside's Inverse Formula. Solution of Boundary Value Problem by Laplace Transform: Heat Conduction, Wave Equation, Transmission Lines
	Unit III – Fourier Integral and Transform: Fourier Integral Theorem, Fourier Sine and Cosine Integral, Fourier's Complex Integral, Fourier Transform, Fourier Sine and cosine Transform, Properties of Fourier Transform, Convolution Theorem, Parseval's Identity (for Fourier Transforms, Sine Transform, Cosine Transform) , Fourier Transform of Derivatives, Relationship between Fourier and Laplace Transforms, Solution of Partial differential Equation by Fourier transform, Fourier Transform of partial derivative of function, Solution of heat conduction problem by Fourier Sine Transform , Finite Fourier Transform, Finite Fourier Sine and Cosine Transform of derivatives.
	Unit IV – Z-Transforms: Introduction, Properties of Z-Transforms: Change of scale, Shifting Property, Linearity, Multiplication by k, Division by k, Initial value, Final value, Partial Sum, Convolution Property, Convolution Property by Casual sequence, Inverse Z-Transform, Inverse Z-Transform By Division, Inverse Z-transform by Expansion and Partial fraction, Inversion by Residue method, Solution of Difference Equation.
	Reference Books: **1.** Advanced Engineering Mathematics, H.K. Dass, S. Chand **2.** Advanced Modern Engineering Mathematics, Glyn James, Pearson **3.** Theory and Problems of Fourier Analysis with Application of Boundary value Problem, Murray R. Spiegel, Shaum's Outline series **4.** Theory and Problems of Laplace Transform, Murray R.Spiegel, Shaum's Outline series **5.** The Use of Integral Transforms: I.N. Sneddon

	Suggested digital platform: NPTEL/SWAYAM/MOOCs

Semester III **Practical-4 Code: MMT3P04** **DSC (Core)** **Credits-2 No. of hours 30**	**Syllabus for Practical-4** **MMT3P04: Python Programming**
	Course Outcomes: CO1: *Students describe basic properties of Forier series, Laplase Transform and Z-transform using python programming.* CO2: *Students implement their knowledge in solving various problems on related concepts and develop the proof of coutably subadditive property.* CO4: *Students determine questions related to Forier series, Laplase Transform and Z-transform.* CO5: *Students will understand its significance and relevance to various mathematical disciplines and its applications in other fields.* CO6: *They will develop teamwork and communication skills while learning from each other's approaches and insight.*
	List of practical problems on Advance Mathematical Method using python Programming: Note: **Minimum 12** topics from listed practical problems must be conducted, at least 3 from every unit. (1) Find the Fourier Series representing f(x)=x, $0 < x < 2\pi$ and sketch its Graph from x= -4π to x = 4π By python Program (2) Find the Fourier series expansion of the periodic function of period 2π , $f(x) = x^2$, $-\pi \leq x \leq \pi$ Hence find the sum of series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \pm \cdots$ by python program. Plot Graph (3) Represent the following function by Fourier sine series $f(t) = \begin{cases} t & 0 < t \leq \frac{\pi}{2} \\ \frac{\pi}{2} & \frac{\pi}{2} < t \leq \pi \end{cases}$. Plot Graph. (4) Obtain the Fourier Cosine Series of expansion of the Periodic function defined by $f(t) = \begin{cases} sin\left(\frac{\pi t}{l}\right) & 0 < t < l \\ 0 & Otherwise \end{cases}$. Plot the Graph. (5) Find Laplace transform of t^n and Generalize the Program of Laplace Transform of t^n at any value of n>0 by Python Programming.

(6) Find Laplace Transform of $e^{at}\sin bt$ and $e^{at}\cos bt$ Generalize the Program of Laplace Transform of $e^{at}\sin bt$ and $e^{at}\cos bt$ any value of a and b by Python Programming.

(7) Prove that Property $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$ for positive integer n and Using this Property Generalize python program Laplace transform of function take input as n and function f(t)=cos5t, sin5t, e^{5t}.

(8) Prove that $L\{f(t)\} = F(s)$, then $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)ds$. Using this property generalize the Python Program for considering f(t)=cos3t, sin3t, $e^{-4t}sin3t$.

(9) Find Laplace Transform of Periodic function $f(t) = e^t$ for 0<t<2π, Using $L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$ and Generalize the Program on Periodic function and find Laplace Transform.

(10) Find the Inverse Laplace Transform of $\frac{s+4}{s(s-1)(s^2+4)}$ by Partial fraction method and generalize by Python Program.

(11) Solve Inverse Laplace Transform $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ by convolution theorem and Generalize Python code to find the value at any a, b.

(12) Using Laplace Transform, find the solution of the initial value problem y''-4y'+4y=64sin2t. Generalize Python Program to plot the graph of solution and write geometrical interpretation.

(13) Using Laplace Transform, solve $\frac{dx}{dt} + y = 0$ and $\frac{dy}{dt} - x = 0$ under the condition x(0) =1,y(0)=0. Plot the graph and interpret the solution.

(14) Find the Inverse Laplace Transform of $\frac{1}{(s+1)(s^2+1)}$ by Residue method. Generalize Python Program and plot the graph of solution and give geometrical interpretation of solution.

(15) Find Fourier sine and cosine Transform of $f(x) = e^{-2x} + 4e^{-3x}$. Generalize Python Program.

(16) Finite Fourier Sine and cosine Transform of:

(i) $f(x) = 1$ in $(0, \pi)$

(ii) $f(x) = x$ in $(0, l)$

(iii) $f(x) = x^2$ in $(0, l)$

(iv) $f(x) \begin{matrix} = 1 & in\ 0 < x < \frac{\pi}{2} \\ = -1 & in\ \frac{\pi}{2} < x < \pi \end{matrix}$

(17) Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$,$0 < x < 4$, $t > 0$, given $u(0,t) = 0; u(4,t) = 0$, $u(x,0) = 3\sin\pi x - 2\sin 5\pi x$ Generalize the program. Plot the graph and interprete the result.

(18) Find Z-transform of $c^k\, cosh\alpha k$, $sin\alpha k$ and $cos\alpha k$ and generalize a program.

	(19) Using Z -Transform Solve the difference equation and a python program $y_{k+1} - 2y_{k-1} = 0, k \geq 1, y_{(0)} = 1$ (20) Using the property $Z[kf(k)] = -z\frac{d}{dz}F(z)$, find Z-transform of k and generalize the program. (21) Find Inverse Z-transform of $\frac{9z^3}{(3z-1)^2(z-2)}$ by Residue method and generalize the python program. **Reference Books:** **1.** Advanced Engineering Mathematics, H.K. Dass, S. Chand. **2.** Advanced Modern Engineering Mathematics, Glyn James, Pearson **3.** Theory and Problems of Fourier Analysis with Application of Boundary value Problem, Murray R. Spiegel, Shaum's Outline series. **4.** Theory and Problems of Laplace Transform, Murray R. Spiegel, Shaum's Outline series **5.** The Use of Integral Transforms: I.N. Sneddon. Suggested digital platform: NPTEL/SWAYAM/MOOCs.

Semester III **Paper – IV** **M13:(A)** **Code: MMT3T13** **DSE (Elective 3)** **Credits-4** **No. of hours 60**	Course Title: **GENERAL THEORY OF RELATIVITY** **Course Outcomes:** CO1: ***Foundational Knowledge****: Students will be able to update their basics of Tensors and learn different types of tensors with their applications.* CO2: ***Elementary Skills****: Students will be able to understand the approximation theories of Newton's equations to Poisson equations and vice-versa* CO3: ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving.* CO4: ***Application****: By applying the principles of basic tools, like Christoffel symbols, through the course curriculum, students can solve a variety of practical problems in cosmology.*
	Syllabus for M.Sc. Semester – III **Paper - M13 (A): General Theory of Relativity** **MMT3T13**
	Unit I – Tensors and Christoffel symbols: Tensor Algebra, Riemannian geometry, Christoffel Symbols of first and second kind, Curvature Tensor: Covariant Curvature tensor, Ricci tensor, Energy

	Momentum Tensor, Einstein Tensor, The Bianchi identity.
	Unit II - Approximations to field equations: The principle of covariance, The principle of equivalence, Geodesic principle, Newton's equations of motion as an approximation of geodesic equations, Poisson's equations as an approximation to Einstein field equations.
	Unit III – Metric and Properties of universe in Gravitational field (1): Gravitational field equations in free space, Exterior Schwarzschild's solution and its isotropic form, Birkhoff's theorem, Schwarzschild singularity, Equation of planetary orbit, Advance of Perihelion of a planet, Bending of light rays in the gravitational field, Gravitational Red shift in the spectral lines.
	Unit IV -Metric and Properties of universe in Gravitational field (2): Newtonian Incompressible star, The pressure contribution mass of static, spherically symmetric System, The Tolman-Oppenheimer-Volkoff (TOV) Equation, Schwarzschild's Interior solution, Linearization of the field equations.
	Reference Books: **1.** Introduction to General Relativity: Ronald Adler, Maurice Bezin and Manamen Schiffer, McGraw-Hill Kogakusha Ltd. **2.** Lecture Notes on General Theory of Relativity, Øyvind Gron (Oyvind Gron), Springer publication. Unit 4: Chapter 10, articles [10.1, 10.2,10.3, 10.4] **3.** Lecture on General Relativity, T M Karade, G S Khadekar and Maya S Bendre, Sonu Nilu Publication (2004) **4.** Introduction to theory of relativity, Rosser W.G.V., ELBS (1972). **5.** Relativity Special, General and Cosmology, Rindler W., Pub. Oxford University Press (2003). **6.** The Classical Theory of Fields by Landau I.D. and Lifshitz E.M., Pub. Pergamon Press (1978). Suggested digital platform: NPTEL/SWAYAM/MOOCs

Semester III **Paper IV-M13:(B) Code: MMT3T13** **DSE (Elective 3)**	Course Title: **FLUID DYNAMICS** **Course Outcomes:** CO1: ***Foundational Knowledge:*** *Students will develop a profound understanding of the basic ideas of fluid velocity, streamlines, vortex motion, rotational and irrotational flows. Students will also get know about interrelating these concepts with one another and draw distinction between local and particle rates of change* CO2: ***Elementary Skills****: Students will learn about Sinks, Sources and doublets,*

Credits-4 **No. of hours 60**	*Milne Thomson Circle theorem and Blasius theorem. They will get to know about two- and three-dimensional inviscid fluid flows* CO3: ***Basic Analytic skills****: Students will study and understand inviscid fluid flow and use the continuity equation to determine whether an inviscid flow is incompressible. Students will gain analytical skill and analyse properties of various fluid flows.* CO4: ***Application****: Students will get a deep understanding of fluid flow behaviours and will be able to think critically and apply the knowledge in the study of analysis of fluid motion. This course will also make the students able to prove the results related to Fluids Dynamics.*
	Syllabus for M.Sc. Semester – III **Paper-M13(B): Fluid Dynamics** **MMT3T13**
	Unit I Real Fluids and Ideal Fluids. Velocity of a Fluid at a Point. Stream Lines and Path Lines. Steady and Unsteady Flows. Velocity Potential. Velocity Vector. Local and Particle Rate of Change. Equation of Continuity. Acceleration of a Fluid. Condition at a Rigid Boundary. General Analysis of Fluid Motion. Euler's Equation of Motion. Bernoulli's Equation. Worked Examples. Discussion of the Case of Steady Motion under Conservative Body Forces. Some Further Aspects of Vortex Motion.
	Unit II Sources, Sinks and Doublets. Images in a Rigid Infinite Plane. Images in Solid Spheres. Axisymmetric Flows. Stokes' Stream Function. The Complex Potential for Two-dimensional Irrotational, Incompressible Flow. Complex Velocity Potential for Standard Two-dimensional Flow: Uniform stream, Line source and Line sink, Line doublets, Line vortices. Two-dimensional image systems. The Milne-Thomson Circle Theorem. Some applications of Circle Theorem. Extension of Circle Theorem. The Theorem of Blasius.
	Unit III The Equations of State of a Substance, The First Law of Thermodynamics, Internal Energy of a Gas, Functions of State, Entropy, Maxwell's Thermodynamic Relation, Isothermal Adiabatic and Isentropic Processes. Compressibility Effects in Real Fluids. The elements of wave motion: One Dimensional Wave Equation, Wave Equation in Two and Three Dimensions, Spherical Waves, Progressive and Stationary Waves.

	Unit IV The Speed of Sound in a Gas, Equation of Motion of a Gas. Sonic, Subsonic, Supersonic Flows; Isentropic Gas Flow. Reservoir Discharge Through a Channel of Varying Section: Investigation of Maximum Mass Flow Through a Nozzle. Shock Waves: Formation of Shock Waves, Elementary Analysis of Normal Shock Waves.
	Text Book: F. Chorlton, Text book of Fluid Dynamics, CBS Publishers, Delhi 1985
	Reference Books: **1.** G.K. Batchelor, An Introduction to fluid Mechanics, Foundation Books, New Delhi 1994 **2.** M.D. Raisinghania, fluid Mechanics, S. Chand and Company, Delhi Suggested digital platform: NPTEL/SWAYAM/MOOCs

<table>
<tr><th colspan="2">M.Sc. Semester IV (MATHEMATICS)</th></tr>
<tr><td>Semester IV
Paper I-
M14

Code:
MMT4T14

DSC (Core)

Credits-4
No. of hours
60</td><td>Course Title: DYNAMICAL SYSTEM

Course Outcomes:

CO1: Foundational Knowledge: Students will learn the basic concepts related to Dynamical System. They will also gain the knowledge about The Flow of a Differential Equation and understand their global dynamics.

CO2: Elementary Skills: Students will study about the nonlinear phenomena in physical systems by using a basic knowledge. They will be Capable of determining fixed points and their stability and solve the mathematical problems.

CO3: Basic Analytic skills: Students will study about limits sets, flow boxes, Poincare Bendixson Theorem and its application . Students will gain analytical skill to describe the time evolution of systems which arise from biology and others subjects.

CO4: Application: Students will be able to think critically about how autonomous and non autonomous Differential equation are differentiated. They will be apply the knowledge of Dynamical systems in the study of analysis using mathematical Concepts and techniques.</td></tr>
<tr><td rowspan="5"></td><td>Syllabus for M.Sc. Semester – IV
Paper - M14: Dynamical System
MMT4T14</td></tr>
<tr><td>Unit I: Dynamical systems and vector fields. The Fundamental Theorem. Existence and uniqueness. Continuity of solutions in Initial Conditions. On Extending Solutions. Global Solutions. The Flow of a Differential Equation.</td></tr>
<tr><td>Unit II: Non-linear Sinks. Stability. Liapunov Function. Gradient Systems. Gradients and Inner Products.</td></tr>
<tr><td>Unit III: Limit sets, Local Sections and flow boxes, Monotone Sequences in Planar Dynamical Systems. The Poincare Bendixson Theorem. Applications of Poincare Bendixson Theorem; Ecology: One Species, Predator and Prey, Competing Species.</td></tr>
<tr><td>Unit IV: Asymptotic Stability of Closed Orbits, Discrete Dynamical Systems. Stability and Closed Orbits. Non Autonomous Equations and Differentiability of Flows. Persistence of Equilibria, Persistence of Closed Orbits. Structural stability.</td></tr>
<tr><td>Text Book:
Differential equations, dynamical systems & linear algebra: M.W. Hirsch & S. Smale, Academic Press, 1975.</td></tr>
</table>

	Reference Book: Dynamical systems: V.I. Arnold, Springer Verlag, 1992. Suggested digital platform: NPTEL/SWAYAM/MOOCs

Semester IV **Paper – II** **M15** **Code: MMT4T15** **DSC (Core)** **Credits-4** **No. of hours 60**	Course Title: **MEASURE AND INTEGRATION THEORY** **Course Outcomes:** CO1:***Foundational Knowledge****: Students will be able to understand the fundamental concept of measure and Lebesgue measure. Students should be able to understand the construction of the Lebesgue integral and know its key properties.* CO2: ***Elementary Skills****: Students will be able to investigate fundamental concepts of measure and integration theory and apply the definition and properties of Lebesgue Measure and measurable sets.* CO3: ***Basic Analytic skills****: The main outcome of the course is to equip students with necessary basic analytic skills for problem solving with various types of measurable sets, measurable functions, and solution to a problem of the Lebesgue integral of the bounded measurable functions, unbounded measurable function and integral of a non-negative functions.* CO4: ***Application****: By applying the principles of basic tools through the course curriculum, students can solve a variety of practical problems involving borel measurability sets, measurable functions, Riemann and Lebesgue integral.*
	Syllabus for M.Sc. Semester – IV **Paper - M15: Measure and Integration Theory** **MMT4T15**
	Unit I- Lebesgue Measure of a Set: Length of an interval, Length of a set, σ- rings of sets, σ-algebra of sets, Measure of a set, Measure of an Open Set and a Closed Set, Measure of an Open Interval, Measure of a Closed Interval, Measure of Rectangle, Measure of a Parallelopiped, Outer Measure, Exterior and Interior Measure (Lebesgue Measurable set), Almost Everywhere, Cantor's Tennary Set, Limiting Sets, Covering in the Sense of Vitali, Set of the Type F_σ, Set of the Type G_δ, Borel Set, Borel Measurable.
	Unit II- Measurable Functions: Measurable Functions, Properties of Measurable Functions, Operations of Measurable Functions, Almost Everywhere, Equivalent Functions, Characteristic Functions, Simple Functions, Step Function, Limit Superior and Limit Inferior, Lebesgue Measurable Functions, Borel Measurability Functions, Little Wood's Three Principles.
	Unit III- The Lebesgue Integral: General Definition of Lebesgue Integral of a Function, To Define Lebesgue

	Integral, To Define Lebesgue Integral of an Unbounded Funtion f(x) Define Over a Measurable Set E , (Integral of Non-Negative Function), General Lebesgue Integral, Bounded Convergence Theorem, Fatou's Lemma.
	Unit IV- Theorems of Convergence of Sequences of Measurable Functions: Convergence in Mean, Convergence in Measure, Pointwise Convergence, Convergence Almost Everywhere, Uniform Convergence.
	Reference Books: **1.** Principles of Mathematical Analysis, Third Edition, Walter Rudin, McGrawc Hill Education (India) Private Limited. **2.** Real Analysis, H. L. Royden, P. M. Fizpatrick, Fourth Edition, Pearson Education Asia Limited and China Machine Press. **3.** Measure and Integration, Goyal Gupta, Edited by S. S. Gupta & G. S. Gupta, Edition 2021, Krishna Publication. **4.** Mathematical Analysis, Fourth Edition, S. C. Malik, Savita Arora, New Age International (P) Limited, Publishers. **5.** Real Analysis, V. karunakaran, Pearson. Suggested digital platform: NPTEL/SWAYAM/MOOCs

Semester IV **Paper – III** **M16** **Code: MMT4T16** **DSC (Core)** **Credits-4** **No. of hours 60**	Course Title: **PARTIAL DIFFERENTIAL EQUATIONS** **Course Outcomes:** CO1: ***Foundational Knowledge****: Students will be able to classify partial differential equations upon their linearity and find corresponding integral surfaces.* CO2: ***Elementary Skills****: Students will be able to solve linear partial differential equations of both first and second order.* CO3: ***Basic Analytic skills****: Students will be able to classify partial differential equations and transform second order partial differential equations into canonical form.* CO4: ***Application****: Students will be able to solve boundary value problems for Laplace's equation, the heat equation, the wave equation by separation* of variables, in Cartesian, polar, spherical and cylindrical coordinates.
	Syllabus for M.Sc. Semester – IV **Paper - M16: Partial Differential Equations** **MMT4T16**
	Unit I – Linear First order partial differential equations: Curves and surfaces, First order Partial Differential Equations, classification of

	first order partial differential equations, classifications of Integrals, Linear equations of first order. Pfaffian differential equations, Criteria of Integrability of a Pfaffian differential equation. Compatible systems of first order partial differential equations.
	Unit II - Semi-linear, Quasi-linear and non-linear partial differential equations: Charpit's method, Jacobi method of solving partial differential equations, Integral surfaces through a given curve for a linear partial differential equations: Cauchy Problem, Quasi Linear Equations: Geometry of Solutions, Non-linear First Order partial differential equations.
	Unit III – Second order Partial Differential Equations (Wave and Heat equations): Second order Partial Differential Equations, Classification of second order partial differential equation, Vibration of an infinite string (both ends are not fixed), Physical Meaning of the solution of the wave equation. Vibration of a semi-infinite string, Vibration of a string of finite length:(Method of separation of variables), Uniqueness of solution of wave equation. Heat conduction Problems with finite rod and infinite rod. Uniqueness of solution of heat equation.
	Unit IV – Second order Partial Differential Equations (Laplace equation): Laplace equation, Boundary Value Problems: Dirichlet's problems and Neumann problems, Maximum and minimum principles . Dirichlet Problems and Neumann problems for a circle, for a rectangle and for an upper half plane. Families to equipotential surfaces, Solution of Laplace equation, Laplace equation in polar form, Laplace equation in spherical polar coordinates. Kelvin's inversion theorem, Stability theorem, Duhamel's Principle.
	Reference Books: **1.** T. Amarnath: An elementary course in Partial differential equations, 2nd edition, Narosa publishing House (2012). **2.** Mark Pinsky: Partial differential equations and boundary-value problems with applications, AMS,3rd edition(2011). **3.** I. N. Sneddon: Elements of Partial Differential Equations, McGraw Hill Int.

	4. Fritz John: Partial Differential Equations, Springer(1952). Suggested digital platform: NPTEL/SWAYAM/MOOCs

Semester IV **Paper – IV** **M17** **Code: MMT4T17** **DSE (Elective 4)** **Credits-4** **No. of hours 60**	Course Title: **COSMOLOGY** **Course Outcomes:** CO1: ***Foundational Knowledge:*** *Student will be able to update their basis knowledge about cosmology studies. How the history of universe led to the stars, galaxies, and other features we can observe today.* CO2: ***Elementary Skills:*** *Student will able to study of cosmology. Cosmology is the study of outer space or the universe came to be, what its structure is like, and what the future may hold.* CO3: ***Basic Analytic Skills:*** *The main outcome of the course is to equip student to develop techniques to solve cosmological problem to find the new model.* CO4: ***Application:*** *Cosmology helps to understand how the universe began and how it has evolved over the nature of the universe.*
	Syllabus for M.Sc. Semester – IV **Paper - M17:(A) Cosmology** **MMT4T17**
	Unit I Static cosmological models of Einstein and de Sitter and their derivation and its Properties: (i) The geometry of the Universe (ii) Density and pressure (iii) Motion of test particle (iv) Doppler shift (v) comparison with actual universe, Comparison between Einstein and de-Sitter models.
	Unit II Cosmological principle, Hubble law, Weyl's postulate, Derivation of Robertson Walker Metric and its properties, Motion of a particle and light rays in FRW model, Red shift, Deceleration parameter and Hubble's constant, Matter Dominated era.
	Unit III Friedman Model, Fundamental equation of dynamical cosmology, density and pressure of the present universe, Matter dominated era of the universe, critical density, flat, closed and open universe, age of the universe.
	Unit IV Steady state cosmology, Distance measure in cosmology, Comoving distance, Apparent luminosity and luminosity distance, Relation between Present Density, Age and Hubble's Rate, Angular diameter and Lookback time, Horizons and the Hubble radius; Galaxy count**,** the Particle horizons, the Event Horizon.

	Text Books: **1**. Relativity, Thermodynamics and Cosmology: Richard C. Tolman, Oxford Press **2**. Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity by Steven Weinberg
	Reference Books: **1.** General Relativity and Cosmology, By S. K. Srivastava. **2.** The Classical Theory of Fields, By Landau I.D. and Lifshitz E.M., Pub. Pergamon Press (1978). **3.** Lecture on General Relativity, Sonu Nilu Publication (2004) by T M Karade, G S Khadekar and Maya S Bendre. **4.** The Theory of Relativity Moller C, Pub. Oxford University Press (1982). **5.** Introduction to theory of relativity, Rosser W.G.V., ELBS (1972). **6.** Relativity Special, General and Cosmology, Rindler W., Pub. Oxford University Press (2003). **7.** Relativity: The General Theory, Synge J.L., North Holland Pub. Comp. (1971).

Semester IV **Paper – IV** **M17** **Code: MMT4T17** **DSE (Elective 4)** **Credits-4** **No. of hours 60**	Course Title: **NUMBER THEORY** **Course Outcomes:** CO1: ***Foundational Knowledge:*** *Student will be able to update their basis of divisibility and modular arithmetic.* CO2: ***Elementary Skills:*** *Student will able to handle theory of congruence, understand Mobius inversion formula, Euler's theorem.* CO3: ***Basic Analytic Skills :*** *The main outcome of the course is to equip student to develop techniques to solve congruence problem for degree one and some special cases of degree two, using Chinese remainder theorem* CO4: ***Application:*** *Number theory is applied in Cryptography, Device authentication, Coding, websites for Ecommerce, Security system, many more .*
	Syllabus for M.Sc. Semester – IV **Paper – M17 (B): Number Theory** **MMT4T17**
	Unit I- Divisibility: Introduction , Definition of divisibility, Division Algorithm, The Greatest Common Divisor, Least Common Multiple , The Euclidean Algorithm , Primes , Greatest Common Divisor and Least Common Multiple through Prime Factorization, The Linear Diophantine Equations $ax + by = c$.

	Unit II- Congruences : Congruence , Basic Properties of Congruence, Decimal Representation of Integers, Solution of congruence, Chinese Remainder Theorem, The Fermat theorem , Fermat Little theorem and pseudo Primes, Wilson's Theorem .
	Unit III Euler's Generalization of Fermat's theorem: Sum and Number of divisors, The Mobius Inversion Formula, The greatest Integer Function, Euler's Phi- Function, Euler's theorem, Properties of Phi function.
	Unit IV Primitive roots, Primitive roots for Primes, Polynomial congruence, The congruence $x^2 \equiv a(mod\ p)$, General quadratic congruence, Quadratic residues.
	Reference Books: **1.** Elementary Number Theory, David M. Burton, Tata McGRAW- HILL, 2006. **2.** An Introduction to Theory of Number, Niven, H. Zuckerman and H.L. Montgomery, 5^{th} Edition. **3.** Classical Introduction to Modern Number Theory, K. Irelandand M. Rosen, A, Springer (Second Edition) **4.** A concise Introduction to the Theory of Numbers, A Baker, Cambridge University Press, 1984. **5.** A course in arithmetic- GTM Vol.7, J.P. Serre, Springer Verlag, 1973.